Critical current under an optimal time-dependent polarization direction for Stoner particles in spin-transfer torque induced fast magnetization reversal

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Fast magnetization reversal of uniaxial Stoner particles by spin-transfer torque due to the spin-polarized electric current is investigated. It is found that a current with a properly designed time-dependent polarization direction can dramatically reduce the critical current density required to reverse a magnetization. Under the condition that the magnitude and the polarization degree of the current do not vary with time, the shape of the optimal time-dependent polarization direction is obtained such that the magnetization reversal is the fastest.

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The advent of miniaturization and fabrication of magnetic particles at nano-meter scale[1] (called Stoner particles) makes the Stoner-Wohlfarth (SW) problem[2] very relevant to nano-sciences and nano-technologies. For a Stoner particle, strong exchange interactions keep the magnetic moments of atoms rigid such that the constituent spins rotate in unison. The magnetization dynamics can be manipulated by a laser light[3], or a magnetic field[4, 5, 6], or a spin-polarized electric current[7, 8, 9, 10, 11, 12, 13, 14, 15]. Among them, magnetization manipulation[2] of Stoner particles by electric current of spin-polarized electrons is of great current interests in spintronics because of its locality and low power consumption. The idea of spin-transfer torque (STT) generated by a spin-polarized electric current was independently suggested[7] by Slonczewski and Berger in 1996, and was verified by several experiments[10]. Important issues in its applications are to lower the critical current required to reverse a magnetization[13] and to design a current pulse such that the magnetization can be switched from one state to another fast. Many reversal schemes[6, 11, 12, 13, 14, 15] have been proposed and examined. Ideas include thermal-assistance[11, 12] and sample designs[13, 14, 15]. However, the question of how much the critical current can further be lowered is still unknown. In this report, the optimal time-dependence of the current polarization direction for the fast magnetization reversal is derived when the magnitude and the polarization degree of the current is fixed. It is shown that the critical current density can be dramatically reduced if the optimal time-dependent polarization is employed.

The prototype of STT systems is a magnetic multilayered structure of nano-meter scale as illustrated in Fig. 1. It consists of two ferromagnets which are sandwiched among three nonmagnetic metallic layers. Electrons travel along the $+\hat{z}$ direction (from the left to the right) in the sample (the direction of the current is opposite as shown in Fig.1). The first ferromagnet F1 is usually very thick so that the current does not affect the magnetization $\vec{M}_1$ of F1. Electrons are polarized along $\vec{M}_1$ after they pass through F1, and remain their polar-

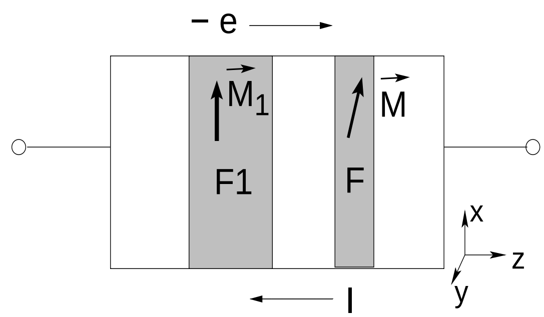

FIG. 1: Schematic illustration of the STT structure. Note that the direction of the electrical current is opposite to that of electron flow.

izations before entering the second ferromagnet F when the thickness of the spacer layer between F1 and F is much smaller than the spin diffusion length. The polarized electrons transfer their spin angular momenta to F, resulting in so-called STT[7]. This STT can affect the dynamics of magnetization $\vec{M}$ of F when its thickness is thin enough. Theoretical studies[7, 8, 9] show that the STT Γ is proportional to the current with the following form

$$\Gamma \equiv [\frac{d(\vec{M}V)}{dt}]_{STT} = \frac{\gamma\hbar I}{\mu_0 e} g(P, \vec{m}\cdot\hat{s})\vec{m}\times(\vec{m}\times\hat{s}) \quad (1)$$

where $\vec{m}, \hat{s}$ are the unit vectors of $\vec{M}$ and the polarization direction (along $\vec{M}_1$) of the current, respectively. In the expression, V, $\hbar$, μ_0, and e denote the volume of F, the Planck constant, the vacuum magnetic permeability, and the electron charge, respectively. $\gamma = 2.21\times10^5(rad/s)/(A/m)$ is the gyromagnetic ratio. The exact microscopic formulation of the STT is still a subject of study and debate[8, 9]. Different theories differ themselves in different expressions of function g that depends on the degree of the polarization P of the current and relative angle between $\vec{m}$ and $\hat{s}$. All experimental investigations[10] so far are consistent with the result of Slonczewski[7] which will be used throughout this study,

$$g(P, \vec{m}\cdot\hat{s}) = \frac{4P^{3/2}}{(1+P)^3(3+\vec{m}\cdot\hat{s}) - 16P^{3/2}}. \quad (2)$$

The magnetization dynamics of a Stoner particle (F in Fig. 1) under an effective magnetic field $\vec{H}_t$ and a polarized current is governed by the so-called modified Landau-Lifshitz-Gilbert (LLG) equation[8] with an additional term (the third term on the right of the equation below) due to the STT of Eq. (1)

$$\frac{d\vec{M}}{dt} = -\gamma\vec{M}\times\vec{H}_t + \alpha\vec{m}\times\frac{d\vec{M}}{dt} + \gamma a_I\vec{M}\times(\vec{M}\times\hat{s}), \quad (3)$$

where $a_I = \hbar I g/(\mu_0 e M^2 V)$ is a dimensionless parameter, and α is a phenomenological dimensionless damping constant whose typical value ranges from 0.01 to 0.22 for Co films[4]. Because the magnitude of $\vec{M}$ does not change with time for a Stoner particle, $\vec{M}$ can be described by the polar angle θ and the azimuthal angle ϕ in the spherical coordinate, and Eq. (3) can be rewritten in a dimensionless form

$$(1+\alpha^2)\frac{d\vec{m}}{dt} = -\vec{m}\times\vec{h}_1 - \vec{m}\times(\vec{m}\times\vec{h}_2), \quad (4)$$

where

$$\vec{h}_1 = \vec{h}_t + \alpha a_I\hat{s},$$
$$\vec{h}_2 = \alpha\vec{h}_t - a_I\hat{s}.$$

t in Eq. (4) is measured in unit of $(\gamma M)^{-1}$. The magnetization and the magnetic field are in the units of M. the total field $\vec{h}_t = \vec{h}+\vec{h}_i$ includes both the applied magnetic field $\vec{h}$ and the internal field $\vec{h}_i$ due to the magnetic anisotropic energy density $w(\vec{m})$, $\vec{h}_i = -\nabla_{\vec{m}}w(\vec{m})/\mu_0$. Different particles are characterized by different magnetic anisotropy. We consider only the uniaxial particles of magnetic anisotropy $w(\vec{m}) = -km_z^2/2$ with its easy axis along the z-direction so that and $\vec{h}_i = -\frac{\partial w(\cos\theta)}{\partial(\cos\theta)}\hat{z} \equiv f(\cos\theta)\hat{z}$. Let $\hat{e}_r$, $\hat{e}_\theta$, $\hat{e}_\phi$ be the three spherical unit vectors of $\vec{m}$, so $\vec{h}_i = -k\sin\theta\cos\theta\hat{e}_\theta + k\cos^2\theta\hat{e}_r$. In terms of θ and ϕ, Eq. (4) can be written as

$$\begin{aligned}(1+\alpha^2)\dot{\theta} &= h_{t,\phi} + \alpha h_{t,\theta} + a_I(\alpha s_\phi - s_\theta),\\ (1+\alpha^2)\sin\theta\dot{\phi} &= \alpha h_{t,\phi} - h_{t,\theta} - a_I(\alpha s_\theta + s_\phi).\end{aligned} \quad (5)$$

Here $h_{t,\theta}$, $h_{t,\phi}$ and s_θ, s_ϕ, s_r are the $\hat{e}_\theta$, $\hat{e}_\phi$, and $\hat{e}_r$ components of $\vec{h}_t$ and $\vec{s}$, respectively.

Because $\vec{h}_1$ and $\vec{h}_2$ are in general non-collinear, the dynamics with the additional STT term in Eq. (4) is very different from that without this term which describes a Stoner particle only in a magnetic field. The particle energy can only decrease in a static magnetic field since the field cannot be an energy source[6]. However, a polarized electric current can pump energy into the particle through the STT. Thus, the STT term from even a dc current can be an energy source, and the dynamical behavior of a Stoner particle under an STT is much richer than its counterpart in a static magnetic field. According to Eq. (4), the magnetization undergoes a precessional motion around field $\vec{h}_1$ and a damping motion toward a different field $\vec{h}_2$.

The magnetization switching problem for a uniaxial Stoner particle is as follows: In the absence of both polarized current and $\vec{h}(=0)$, $\vec{M}$ in F has two stable directions, $\theta = 0$ and $\theta = \pi$, along its easy axis (z-axis). The goal is to reverse the initial state (say $\theta = 0$) to the target state, $\theta = \pi$, fast by a small polarized electric current. All studies so far assumed that the current polarization direction does not vary with time. However, previous studies[6] on magnetic-field induced magnetization reversal show that a switching field can be dramatically reduced if the direction of the field varies properly with time during a reversal. Thus, it is natural to investigate whether one can use a current with a proper time-dependent polarization direction to lower the critical reversal current density. The precise issue studied here is: For a given system in which damping constant α, anisotropy $f(\cos\theta)$ and the external magnetic field are fixed, the polarized electric current can vary its polarizations direction during a reversal process under the constrain of the constant magnitude I and constant polarization degree P of the current. The issue is to minimize the critical current density, and to find the best shape of polarization direction for the shortest reversal time.

To simplify the calculations, we consider the case of zero external magnetic field, $\vec{h} = 0$. However, the basic idea should directly be applicable for non-zero static external field because $\vec{h}$ can be added to $\vec{h}_i$. Then Eq. (5) becomes,

$$\begin{aligned}(1+\alpha^2)\dot{\theta} &= a_I(\alpha s_\phi - s_\theta) - \alpha f(\cos\theta)\sin\theta,\\ (1+\alpha^2)\sin\theta\dot{\phi} &= -a_I(\alpha s_\theta + s_\phi) + f(\cos\theta)\sin\theta.\end{aligned} \quad (6)$$

$\hat{s} = \hat{s}(t)$ is in general a function of the time. Different function will lead to different angular velocities for θ and ϕ. Thus, the magnetization reversal time from the initial state ($\theta = 0$) to the target state ($\theta = \pi$), defined as $T \equiv \int_0^\pi d\theta/\dot{\theta}$, depends on $\hat{s}(t)$. In order to find the optimal $\hat{s}(t)$ that minimizes T, one only needs $a_I(\alpha s_\phi - s_\theta)$ or $g(P, s_r)(\alpha s_\phi - s_\theta)$ to be maximum such that $\dot{\theta}$, according to Eq. (6), will be the largest at any θ. This observation is important and it can be applied to other function forms of g. Because $s_r^2 + s_\theta^2 + s_\phi^2 = 1$, the maximum of $g(P, s_r)(\alpha s_\phi - s_\theta)$ can be obtained from the standard Lagrange multiplier method in which one introduces $L \equiv g(P, s_r)(\alpha s_\phi - s_\theta) - \lambda(s_r^2 + s_\theta^2 + s_\phi^2)$. By setting the partial derivatives of L with respect to s_i (i=r, θ, ϕ) to zeros, the maximum of $g(P, s_r)(\alpha s_\phi - s_\theta)$ is

$$[g(P, s_r)(\alpha s_\phi - s_\theta)]_{max} = \sqrt{1+\alpha^2}G(P), \quad (7)$$

where $G(P)=g(P,s_r^*)\sqrt{1-s_r^{*2}}$ and

$$s_r^* = \frac{(1+P)^3}{16P^{3/2}-3(1+P)^3},$$
$$s_\theta^* = -\frac{1}{\sqrt{1+\alpha^2}}\sqrt{1-s_r^{*2}}, \quad (8)$$
$$s_\phi^* = \frac{\alpha}{\sqrt{1+\alpha^2}}\sqrt{1-s_r^{*2}}.$$

Eq. (8) gives the optimal polarization direction which will lead to the shortest switching time under a fixed current magnitude. Although the optimal $\vec{s}^*$ appears to depend only on damping constant α and P, but not on $f(\cos\theta)$, it is in fact time-dependent since $\hat{e}_r$, $\hat{e}_\theta$, $\hat{e}_\phi$ vary with the time. Furthermore, magnetic anisotropy $f(\cos\theta)$ shall influence the evolution of $\vec{m}$ which in turn influences the time-dependence of $\vec{s}^*$. Thus, if they were to change $f(\cos\theta)$ and nothing else, the time-dependent $\vec{s}^*$ would be different.

Under the optimal design of Eq. (8), $\theta(t)$ and $\phi(t)$ will satisfy, respectively,

$$\dot{\theta} = \frac{\hbar I}{\mu_0 e M^2 V}\frac{G(P)}{\sqrt{1+\alpha^2}} - \alpha f(\cos\theta)\sin\theta/(1+\alpha^2), \quad (9)$$

and

$$\dot{\phi} = f(\cos\theta)/(1+\alpha^2). \quad (10)$$

For $w(\vec{m})=-km_z^2/2$, it is straightforward to integrate Eq. (9), and obtain the reversal time T from $\theta=0$ to $\theta=\pi$,

$$T = \frac{2}{k}\frac{(\alpha^2+1)\pi}{\sqrt{4(\alpha^2+1)\hbar^2G^2(P)I^2/(\mu_0 e M^2 V k)^2-\alpha^2}}. \quad (11)$$

In the weak damping limit ($\alpha\to 0$) or large current limit ($I\to\infty$), $T\propto \pi/I$.

For a uniaxial model, the critical switching current or current density J_c can be obtained by setting $\dot{\theta}=0$ in Eq. (9). This is because $\dot{\theta}$ cannot be negative if the magnetization of a uniaxial particle moves from $\theta=0$ to $\theta=\pi$. Therefore, the first term in Eq. (9) must exceed the second term due to magnetic anisotropy for all θ's ($\in[0,\pi]$) in a reversal. This simplicity for a uniaxial model comes from ϕ-independence of Eq. (9). Since Eq. (9) is the largest velocity for an arbitrary θ under the best choice of the polarization direction of a current, the critical current should be the one when the smallest $\dot{\theta}$ (for all θ) is zero. The critical reversal current density in our case is

$$J_c = \frac{\mu_0 e M^2 d}{\hbar G(P)}\frac{\alpha}{\sqrt{1+\alpha^2}}Q. \quad (12)$$

Here the current density is defined as $J=I/A$ with A being the cross-section area and d being the thickness of F. $Q\equiv max\{f(\cos\theta)\sin\theta\}$ for $\theta\in[0,\pi]$. $Q=k/2$ for $w(\vec{m})=-km_z^2/2$. One notices, from the derivation of Eq. (12), that the assumptions of time-independence of I and P are not essential for J_c as long as STT is proportional to I and g in Eq. (1).

To see how much the critical switching current density is reduced, let us compare the critical current density of Eq. (12) with those in the previous schemes where the current polarization direction is fixed. When the polarization direction $\hat{s}$ is parallel to the easy axis of the uniaxial magnet F (the critical current density in the perpendicular case is larger), the critical current density for the same anisotropy as that of Eq. (12) is[7, 13, 14]

$$J_c = \frac{\mu_0 e M^2 d}{\hbar g(P,1)}\alpha k. \quad (13)$$

Fig. 2 is the plot of J_c versus damping constant α for $P=0.4$. The dashed line is that for Eq. (13), and the solid line is the result of our new strategy which saturates to a constant at large α limit. At $\alpha=0.1$, J_c in the new strategy is about one fourth of that given by Eq. (13). The difference between Eqs. (12) and (13) depends on the degree of polarization, P. Fig.3 is J_c vs. P at $\alpha=0.1$. It should be pointed out that zero J_c in Eq. (12) at $P=1$ is an artificial result originated from the divergence of $g(1,x)$ at $x=-1$ in the Slonczewski's theory[7]. This divergence is removed in other formulations of g[9].

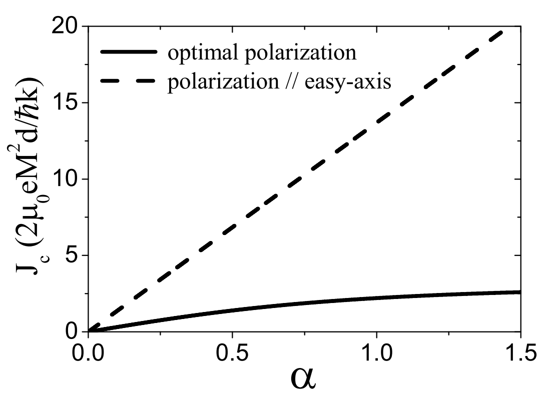

FIG. 2: J_c vs. α for $P=0.4$ and uniaxial model of $w(\vec{m})=-km_z^2/2$.

Although our results are obtained for uniaxial Stoner particles with a specific function of g, the basic ideas and approaches can in principle be generalized to the non-uniaxial cases with other g-functions. Also, almost all experiments so far used a set-up like that illustrated in Fig.1. However, the formulation of an STT induced magnetization reversal does not really rely on the set-up. In general, the same formulation could be applied to a Stoner particle through which a spin current pass. Therefore, advance in generating the pure spin current (with charge current being zero), one of current topics in so-called spin Hall effect, could lead to a new set of experimental set-up beyond the prototype illustrated in

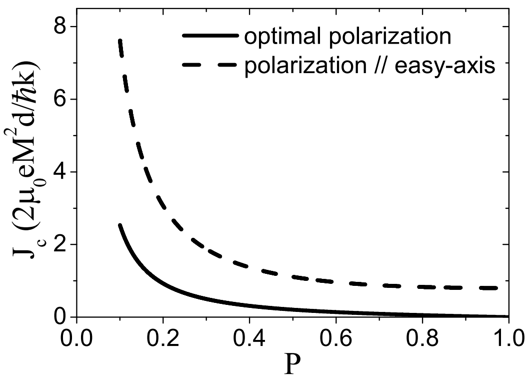

FIG. 3: J_c vs. P at $\alpha = 0.1$. The rest of system parameters are the same as that of Fig. 2.

Fig. 1 in the STT experiments and the STT applications. In the prototype of Fig. 1, it happens that the current of spin-polarized electrons is generated by one of the magnets.

The realization of the results reported here depends largely on our ability in generating a designed spin-polarized electric current. In some sense, the present work converts the issue of critical current to the issue of generating an arbitrary polarized electric current. Any breakthrough in the front of spin current generation shall lead to the great leap forward in magnetization minipulation. One possible way to generate a desired polarized electric current is through controlling magnetization $\vec{M}_1$ of F1 in Fig. 1 by other means. As it was explained early, the polarization direction of the electric current is parallel to $\vec{M}_1$ in an experimental set-up illustrated in Fig. 1. Thus, time-dependence of the polarization of the current is the same as that of $\vec{M}_1$. However, since the response time of electrons to an electric current is usually much faster than that of the magnetization, it may be an experimental challenge to create a required time-dependent spin polarization by the method, especially when the change of polarization is very fast.

In conclusion, we have showed that a proper time-dependent polarized electric current can dramatically reduce the critical current density needed to reverse a magnetization. An optimal time-dependent current polarization is obtained such that the magnetization reversal time is the shortest for uniaxial Stoner particles.

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